Question 1

Find the point(s) on the parabola $y = x^3 - 6x + 1$ where the tangent is parallel to the line $6x - y + 11 = 0.$ Also, determine the equation(s) of the tangent at that point(s).

Let $f(x) = x^3 - 6x + 1$

The derivative is

$f'(x) = 3x^{3-1} - 6x^{1-1} + 0$

$f'(x) = 3x^2 - 6$

The line $6x - y + 11 = 0$ can be expressed as $y = 6x + 11$ so its slope is 6

The slope of the tangent line parallel to $6x - y + 11 = 0$ is also 6.

We must solve $f'(x) = 0$ for x

We have

$3x^2 - 6 = 6$

$3x^2 = 12$

$x^2 = 4$

$x = 2$ or $x = -2$

The points of tangency are $(2, f(2))$ and $(-2, f(-2))$ which correspond to

$(2, -3)$ and $(-2, 5)$

The equation of tangent lines with slope of 6 are

$y - (-3) = 6(x - 2)$ or $y = 6x - 15$

and

$y - 5 = 6(x - (-2))$ or $y = 6x + 17$

Final answers:

Points of tangency: $(2, -3)$ and $(-2, 5)$

Tangent lines: $y = 6x - 15$ and $y = 6x + 17$